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Giant photonic Hall effect in magnetophotonic crystals

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We have considered a simple, square, two-dimensional (2D) PC built of a magneto-optic matrix with square holes. It is shown that using such a magnetophotonic crystal it is possible to deflect a light beam at very large angles by applying a nonzero external magnetic field. The effect is called the giant photonic Hall effect (GPHE) or the magnetic superprism effect. The GPHE is based on magneto-optical properties, as is the photonic Hall effect [B. A. van Tiggelen and G. L. J. A. Rikken, in *Optical Properties of Nanostructured Random Media*, edited by V. M. Shalaev (Springer-Verlag, Berlin, 2002), p. 275]; however GPHE is not caused by asymmetrical light scattering but rather by the influence of an external magnetic field on the photonic band structure.

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Recently, significant attention has been attracted to photonic crystals (PC). The main reason for this interest is the incredible speed with which the results of the investigations are brought into practical use, such as low threshold lasers, guiding and sharp bending of light in photonic crystals, etc. The application of magneto-optical materials in PC introduces new possibilities of light manipulation with a magnetic field. Photons do not possess electric charge and therefore cannot couple to a magnetic field directly. However, a static magnetic field changes the optical properties of a medium, inducing asymmetrical light scattering and light beam bending [1–4]. This effect was called the photonic Hall effect (PHE) [1]. In some sense the photonic Hall effect is caused by a spin-orbit interaction [2–4]; since spin-orbit interactions are small, the photonic Hall effect is small, too.

In the paper, we consider light propagation in magnetophotonic crystals (MPC) [5]. It is shown that the unique properties of photonic crystals make it possible to deflect a light beam at a very large angle by applying magnetic field. The effect is called the giant photonic Hall effect (GPHE). The GPHE is based on magneto-optical properties as is the photonic Hall effect, but the GPHE is not caused by asymmetrical light scattering. Its main mechanism is the influence of the external magnetic field on the photonic band structure of MPC. Although the changes in band structure are small, they may considerably alter the direction of light propagation for some specific conditions. The GPHE is itself similar to a "superprism," in which a small (of the order of a degree or less) variation of the angle of incidence of an electromagnetic wave may result in significant (more than hundreds of degrees) deviation of a refracted wave [6–9].

The GPHE may be observed at a fixed angle of incidence due to the application of the external magnetic field that causes a deviation of the refracted wave at a large angle. Actually, the GPHE and superprism effect are caused not by diffraction but by refraction. The superprism effect in a PC [6–9] can be described briefly as follows. Consider the PC as a diffraction grating representing the surface of the PC that lies on a homogeneous medium imitating the PC. This grating splits the incident wave into several lobes (the Floquet waves). Under proper conditions (frequency, angle of incidence, etc.) it is possible to obtain two nonevanescent lobes: the central lobe and the side lobe. The inhomogeneous nature of the PC results in the appearance of band gaps at fixed direction of wave propagation or in the appearance of certain directions along which the propagation of the Bloch waves is forbidden at a fixed frequency. Matching the inclusion shape and the lattice symmetry, it is possible to achieve the situation where the sidelobe cannot propagate because it points into a forbidden direction. Thus, we have only one wave propagating through the PC, like ordinary refraction. A small variation of the angle of incidence leads to a small change in the direction of propagation of the lobes. The superprism effect is observed if the sidelobe initially directed into the forbidden angle is redirected into an allowed one and the central lobe initially directing the allowed direction is redirected into forbidden one. Thus, the role of the "refracted" wave is now played by the sidelobe and the angle of such a "refraction" changes considerably more than the angle of incidence.

By applying the magnetic field to MPCs, it is possible to switch from one lobe to another by varying the band structure at the fixed angle of incidence. The influence of the external magnetic field on the band structure of the PC with permeable inclusions has been studied previously [10–13]. To the best of our knowledge, the bending of light in MPCs has not been considered in the literature. In the present communication we consider the strong deflection of light that may occur directly in magnetophotonic crystals by applying a magnetic field. By definition this phenomenon is the GPHE.

In optics, the permeability is equal to unity and to control the MPC band structure with magnetic field we deal with magneto-optical materials. The application of magneto-optical materials does not permit a pronounced modification of the band structure due to smallness of the control parameter α (off-diagonal entry of the permittivity tensor). Nevertheless, it is shown that a strong deflection of light (more

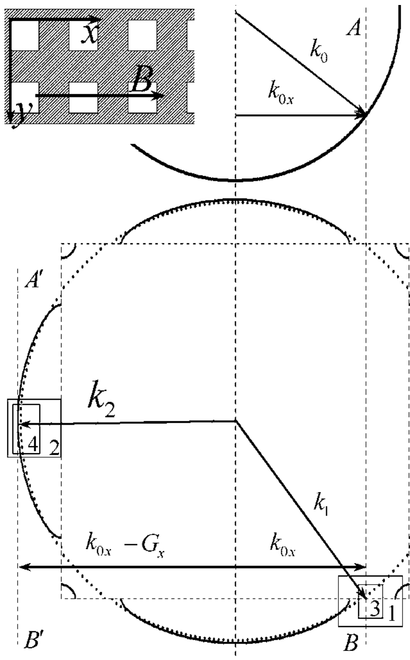

FIG. 1. An outline of refraction of a light beam on the boundary between the vacuum (the upper equifrequency surface) and the photonic crystal (the bottom equifrequency surface). Turning on the external magnetic field, the small variation of the parameter α from 0 to 0.01 results in switching of the solution from k_1 to k_2. $k_0=0.473\,04$, $k_{0x}=0.374\,85$, the angle of incidence $\psi_0=0.9148=52.41°$, $\varepsilon_1=2.5$, $\varepsilon_2=1.5$, the area of the rod's section is one-fourth the cell area. Dashed lines are auxiliary lines representing $k_x=0$, $k_x=k_{0x}$, $k_x=k_{0x}-G_x$ and a boundary of the first Brillouin zone. Solid lines are the equifrequency surface of the TE wave; dotted lines are equifrequency surface of TM wave. The inset shows the topology of our MPC, directions of axes, and external magnetic field.

than 120 deg) may occur directly in MPC by the application of the external magnetic field. This bending happens in a single jump, as it happens in the superprism effect. The value of this jump is almost independent of magnetic field strength and is determined by the structure of the PC cell.

For simplicity we consider a 2D model with the wave vector k having only two nonzero components, namely k_x and k_y [14] (see the insert in Fig. 1).

The MPC under investigation is a square lattice of holes in the magneto-optical matrix. The holes are squares in their cross section and are filled up by a nonmagnetic dielectric with permittivity ε_1. The elementary cell size is equal to a. The vector of the reciprocal lattice $G_x=2\pi/a$. When an external magnetic field B is directed along the x axis, the permittivity tensor for the magneto-optical material is given as follows [15]:

$$\hat{\varepsilon}=\begin{pmatrix}\varepsilon_2 & 0 & 0\\ 0 & \varepsilon_2 & i\alpha\\ 0 & -i\alpha & \varepsilon_2\end{pmatrix}, \qquad (1)$$

where the magneto-optical parameter α is a function of the external magnetic field B_{ext} and is linear with magnetization. We neglect all the effects connected with dissipation by supposing ε_1, ε_2, and α to be pure real values. This assumption is not a principal one for dielectric materials, but for magneto-optical materials such an assumption puts some restrictions on the frequency. We should consider a frequency lying far from resonance and deal with small (about 2×10^{-2}) values of $\alpha\sim B_{ext}$.

The magneto-optical matrix loses its usual symmetry of a 2D structure in a magnetic field. Although the spatial derivatives of the fields along the z direction are still equal to zero, the problem does not reduce to two independent scalar problems. Now the eigenmodes are neither TM nor TE modes but left- and right-polarized ellipsoidal waves. Following the usual approach [16–19] we analyze the light propagation in terms of the equifrequency surface, which appears by a section of the dispersion surface $\omega(k)$ by the plane of ω=const. In the 2D case, if the plane of incidence is perpendicular to the generatrix of the 2D PC, the equifrequency surface is a set of lines on the plane of wave vector k, the Bloch wave vector. To make our argumentation easier we work with an extended band structure [20] without reducing the wave vectors to the first Brillouin zone. The advantage of this approach is in the easier physical interpretation of the results. For example, (i) if the PC contrast $[(\varepsilon_{mat}-\varepsilon_{incl})/\varepsilon_{mat}]$ tends to zero, we arrive at the equifrequency surface of the uniform space; (ii) in many simple cases the angle between the group and phase velocity is less than $\pi/2$ and we do not have to use the concept of backward waves [21].

In our calculations, we employ the E method [22,23] generalized for magneto-optical materials. Fixing the frequency ω, or more exactly the wave number in free space $k_0=\omega/c$, and the incidence angle φ, we find the value of the Bloch wave number. Varying the angle φ, we obtain the entire equifrequency surface.

At zero external magnetic field, the local permittivity tensor has no off-diagonal elements and the MPC becomes a conventional 2D PC with two different eigenmodes, namely TE and TM waves, propagating with different k_B. Thus, the equifrequency surface consists of two sets of curves corresponding to the TE mode and TM mode (Fig. 1).

The external magnetic field alters the equifrequency surfaces only slightly. A local shift of the equifrequency surfaces depends on the angle between the magnetic field and the direction of the wave vector. To understand this behavior, it is worth requesting homogeneous anisotropic magneto-optical materials as a model for waves propagating perpendicular to the optical axis. At a zero magnetic field there are two refracted waves, namely ordinary (TE) and extraordinary (TM). The difference $(k_{or}-k_{ex})$ between the corresponding wave numbers is determined by the anisotropy of the material. For a nonzero magnetic field, the ordinary wave acquires right-hand polarization whereas the extraordinary wave acquires left-hand polarization, or *vice versa*, for the reverse direction of the magnetic field. The changes of the wave numbers are quadratic in α if these changes are less than $(k_{or}-k_{ex})$ and linear in the opposite case. Since in PC the difference $(k_{TE}-k_{TM})$ depends on the direction of propagation (this difference not only changes in value but also can

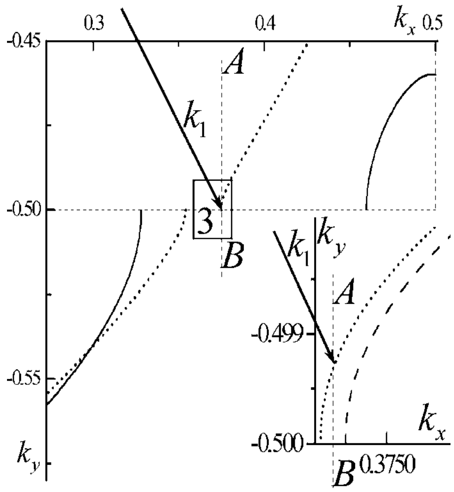

FIG. 2. Zoomed area 1 in Fig. 1. The inset shows the shift of the TM mode in an external magnetic field (zoomed area 3). The solid curve represents the TE wave; the dotted curve represents the TM wave.

change in sign) we observe more complicated behavior of the equifrequency surface of the MPC. To model a MPC we need different uniaxial crystals depending on the direction of wave propagation [24].

Let us consider a particular example of the phenomenon at fixed values of the incidence angle ψ_0 and frequency $k_0 = \omega c$, equal, respectively, to 52.41 deg and $k_0 = 0.473\,04$ (see Fig. 1). Below, all the values of wave numbers are measured in units of $G_x = 2\pi/a$. In particular, $G_x = 1$. The solid curves in Figs. 1–3 present the extended equifrequency surface for a TM mode. The equifrequency surface consists of separate lines because we consider an extended band structure, but the one reduced to the first Brillouin zone, where the equifrequency surface looks like an indissoluble line. The image resolution in Fig. 1 does not allow us to see all the details, so we have selected areas around the solutions (1 and 2) and represented them with different zooming in Figs. 2 and 3 (Fig. 2 corresponds to the first area, Fig. 3 to the second one).

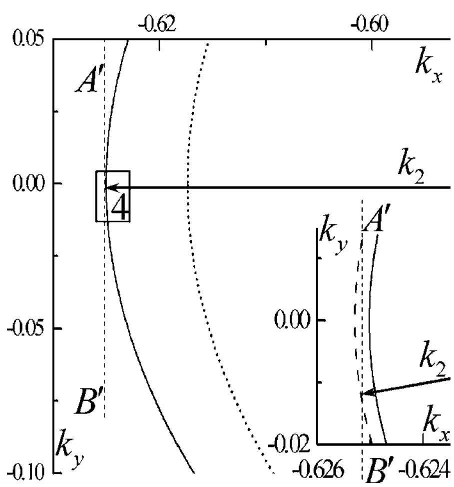

FIG. 3. Zoomed area 2 in Fig. 1. The inset shows the shift of the TM mode in an external magnetic field (zoomed area 4). The solid curve represents the TE wave; the dotted curve represents the TM wave.

Because (see the insert in Fig. 1) the x projection of any wave vector has to be constant to a shift by $\pm nG_x$, where n is an integer and G_x is the x projection of the vector of the reciprocal lattice, the intersections of the vertical dashed lines (corresponding to $k_{0x} \pm nG_x, n \in$ in Figs. 1–3) with the equifrequency surface form the solutions. For a zero external field ($\alpha = 0$), after diffraction on the interface surface of the PC, only the center TM lobe with the wave vector $k_1^{(B=0)}$ can propagate. The sidelobe is found in the forbidden directions: the vertical line $k_{0x} - G_x$ does not intersect the equifrequency surface of the PC (see Fig. 3). All TE lobes lie in the forbidden directions too.

In the presence of the nonzero external magnetic field the equifrequency surface is represented in Fig. 3 and Fig. 2 by dashed lines. We can see that the only propagating lobe has the Bloch wave vector $k_2^{(B)}$, corresponding to the TE sidelobe in the homogeneous PC. The center lobe shifts into a forbidden region. Therefore, turning on an external magnetic field produces "switching" of the direction of light beam propagation from the direction of $k_1^{(B=0)}$ to the direction of $k_2^{(B)}$. This is true for a perfect, infinite crystal where the direction of travel of the "refracted" wave changes by a jump at the appropriate value of applied magnetic field. The value of this jump (120 deg in our case) is almost independent of the magnetic field and is determined by the structure of the PC cell. Regarding sample thickness, as we increase the magnetic field we will observe a gradual increase of the intensity of one solution and decrease of intensity of the other one. The angle between directions of propagation of these solutions still will be the same as in infinite crystal. The thicker the PC slab, the sharper is the exchange from one solution to another.

The scenario described above responds to negative refraction [25] (with respect to the interface surface). Unlike refraction in the Veselago media regarding directions of group and phase velocities, this lobe is a normal (not backward) wave.

It should be noted that in our approach, the angle of incidence and the frequency are fixed. The change in the direction of propagation of the "refracted" wave is achieved by an application of the external field. In contrast to the common superprism effect, the variation of the angle of incidence does not result in any superprism effect. If we decrease the angle of incidence (and decrease k_{0x}) we will have no solution, whereas if we increase the angle of incidence (and increase k_{0x}) we obtain two solutions corresponding to waves propagating in different directions.

Our calculations are illustrative examples only. First of all, we take the high value of the off-diagonal component of the permittivity tensor $\alpha = 2.0 \times 10^{-2}$. Employing the values of Bi:DyIG of $\varepsilon_2 = 5.58$ and $\alpha = 1.98 \times 10^{-3}$ at the same simple geometry results in equifrequency surface shifts of the order of 10^{-6}. This is a demanded accuracy of fixing the angle of incidence. Nevertheless, it is obvious that the shift can be enhanced by employing another geometry and by switching to higher Brillouin zones [26]. By decreasing the volume occupied by a nonmagnetic dielectric, we can enhance the effect up to 4×10^{-5}. Employing the higher Brillouin zones is fraught with negative consequences. Indeed, 2D crystals do not exist in reality, and we can deal with the truncated PC slabs only. As a consequence some of the Bloch waves, in particular those we are dealing with, become leaking waves. To rescue the situation it is possible to sandwich the MPC slab by one-dimensional PC slabs in which the direction of propagation of the leaking waves is forbidden. Fixing all these problems is a subject for a separate study.

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